

# Data Science

## TD 3

### 1. Sampling

**Q. 1** Fourier transform of the Dirac comb:

(a) Use the properties of Fourier transform to compute the Fourier transform of

$$\text{III}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - n)$$

(b) Compute the Fourier transform the following Dirac comb (we define  $\nu_0 = \frac{1}{T}$ ):

$$\text{III}_T(t) = \sum_{n=-\infty}^{+\infty} \delta(t - T n)$$

**Q. 2** Consider the following signal:

$$x(t) = \sin(2\pi \nu_x t + \phi)$$

Compute its Fourier transform  $\hat{x}$  and plot  $|\hat{x}|$ :

**Q. 3** We defined:

$$y(t) = x(t) \text{III}_T(t)$$

Compute the Fourier transform  $\hat{y}$  and plot  $y$  and  $|\hat{y}|$ .

**Q. 4** Redraw the plot when  $x$  is a band-limited signal of spectral width  $B_s$ . (*i.e.*  $\hat{x}(\nu) = 0$  if  $|\nu| > \frac{B_s}{2}$ )

**Q. 5** What will happen if  $\nu_e < B_s$ ?

The sampling of the signal  $y$  is performed by defining  $y[k] = y(kT)$ .

### 2. Reconstructing sampled data

From the samples  $y[k]$ , we can reconstruct the continuous signal  $y(t)$ :

$$y(t) = \begin{cases} y[\frac{t}{T}] & \text{if } \frac{t}{T} \in \mathbb{Z}, \\ 0 & \text{otherwise.} \end{cases}$$

From  $y(t)$ , we obtain a reconstruction  $z(t)$  of the original signal  $x(t)$  by applying a filter with impulse response  $h$ :

$$z(t) = (h * y)(t)$$

**Q. 6** Knowing that  $x$  is a band-limited signal of spectral width  $B_s$ , show that the optimal filter that minimizes the reconstruction error  $E = \int_{-\infty}^{+\infty} |x(t) - z(t)|^2 dt$  is a cardinal sine.

(a) Is this filter causal? What are the issues with such filter?

**Q. 7** We design a filter with the following impulse response:

$$g(t) = \frac{1}{\tau} u(t) e^{-\frac{t}{\tau}}$$

- (a) Compute the transfer function of this filter  $g$ .
- (b) Compute its squared modulus  $|\hat{g}|^2$  and its phase  $\angle(\hat{g})$ ; plot both quantities:
- (c) What is the cut-off frequency  $\nu_c$  of  $g$ :

$$|\hat{g}(\nu_c)|^2 = \frac{1}{2}$$

- (d) What could be the time constant  $\tau$  of this filter to reconstruct  $x$  from  $y$ ?